Influence of Fan and Ducting Characteristics on the Stability of Ground Effect Machines

NORMAN K. WALKER*
Norman K. Walker Associates Inc., Bethesda, Md.

A general incompressible linearized theory is presented for the small heave oscillation of plenum and annular jet ground effect machines (GEM). It is shown that the static stability and damping in heave are both dependent on a system stability parameter that is a function of a jet flow and total pressure rise. Effects of duct losses are also explored and it is shown that system stability improves with a duct loss, but that instability of flow in the ducts can also cause system instability at heave. Results are briefly compared with detailed experimental measurements on a model GEM and are illustrated by examples from current practice.

Nomenclature

b = length of base to inside edge of jets
C = perimeter of base
C_D = discharge coefficient of the plenum chamber
C_P = fan pressure coefficient = total head immediately behind fan/q_t
D = fan diam
F = fan and ducting stability parameter for plenum chamber = (stabilizing if negative) = $(\partial P_t/\partial Q)(Q_0/P_{t_0})/[1 - \frac{1}{2}(\partial P_t/\partial Q)(Q_0/P_{t_0})]$
$\mathfrak{F}$ = fan and ducting stability parameter for annular jet GEM
g = acceleration due to gravity
G = thickness of jet at exit, normal to flow
h = height of jet exit above ground plane
h_0 = height of jet exit above ground plane at equilibrium
h_b = height of base above ground plane at equilibrium
h_σ = effective equilibrium height of jet exits, includes Eames correction
K = damping coefficient
K_s = damping coefficient; jet overfed GEM sinking
K_R = damping coefficient; jet underfed, GEM rising
L = gross lift of machine = $S_b \cdot p_{b_0}$
m = mass of vehicle = W/g
M_y = static pitching moment
$\bar{M}_y$ = $-(\partial M_y/\partial\alpha)/Lb$
p = static pressure of atmosphere
p_b = base pressure relative to atmosphere
p_{b_0} = base pressure relative to atmosphere at equilibrium
P_t = total head at jet exit
P_{t_0} = total head at jet exit at equilibrium
P_m = total head just behind fan
q_m = dynamic head just behind fan
q_t = dynamic head at the tip of the fan due to rotation = $\frac{1}{2}\rho V_t^2$
Q = volume flow through fan
Q_0 = volume flow through fan at equilibrium
S_b = base area
S_j = jet area
V_{e0} = velocity of jet when exhausted to atmospheric pressure
α = pitch angle from horizontal
β = Tulin's stability parameter = $\gamma h_0/h_b\,[(p_{b_0} + p)/p_{b_0}]$, stable if $\beta > 1$
γ = ratio of specific heat for air (γ = 1.4)
ζ = damping ratio
η = diffuser efficiency, i.e., dynamic head recovery/dynamic head at inlet
ρ = mass density of air
λ = jet angle from vertical, positive if directed inwards = fan airflow coefficient = average velocity through fan/tip speed of fan
ξ = function of h, G, and λ = $Q/\rho S_j V_{e0}$
ξ' = $\partial\xi/\partial(h/h_0)$
ψ = a function of h, G, and λ = p_{b_0}/p_t
ψ' = $\partial\psi/\partial(h/h_0)$
ω = undamped frequency in heave
ω_1 = $[g/h_0]^{1/2}$
ω_2 = $[\beta\omega_1]^{1/2}$
Ω = rotational speed of fan

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* President. Associate Fellow Member AIAA.

Introduction

THE ground effect machine derived from many early ideas all over the world, but in this country, at least, the genesis of the current work on these vehicles occurred when Von Glahn[1] of NASA Langley Field, working on vertical takeoff aircraft, showed that the lift of a vertical jet engine was sharply reduced by the presence of the ground, but greatly increased if the jet were annular. This result was due to the static pressure built up inside the nozzle, and gave rise to a series of investigations into the use of the "air cushion" as a lifting device for vehicles moving in close proximity to the ground.

Since the original experiments were made with jet engines, or high-pressure lab air supplies, it was natural for the early workers on performance and stability to use as a basis for their results the thrust of the isolated nozzle, and later to assume either a constant momentum flux or a constant mass flow.

At a much later date, it was realized that this did not represent a satisfactory approximation to a real system, and further work was undertaken assuming that the total head applied to the annular jet was constant. The present paper extends this approach to assume an arbitrary linear variation of total head with mass flow and is deliberately simplified in treatment to highlight the essential problem areas.

Historical Survey

Tulin[2]

The first published account of the dynamics of a GEM in heave is due to Tulin, who examined the case of a thin annular jet machine with compressible flow and constant momentum flux.

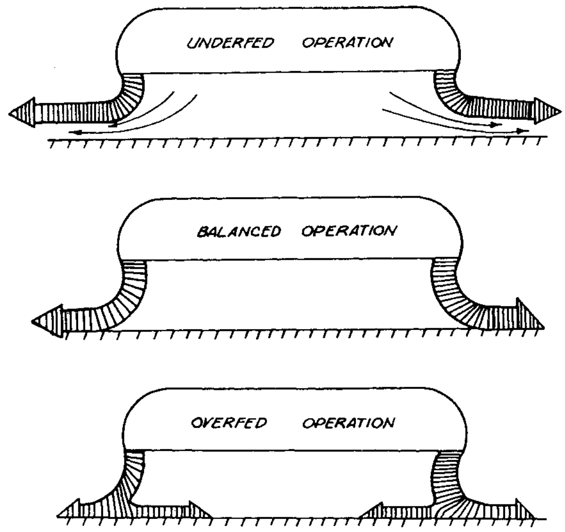

Fig. 1 Jet flow patterns for "underfed," "balanced," and "overfed" operations.

He showed that, in addition to the ordinary "balanced" operation of the jet, two forms of "unbalanced" operation could occur and coined the terms "overfed" and "underfed" to describe these characteristics. In brief, in the case of the "overfed" jet, the momentum flux of the jet is instantaneously greater than that required to support the base pressure. Hence, the jet splits, and part of the jet supplies additional air to the base region. Conversely, the "underfed" jet is too weak to support the base pressure, and base cavity air is forced out below the jet (Fig. 1).

Obviously both these conditions are only transient; the supply of air to the cavity will rapidly raise the base pressure in the case of the "overfed" jet, where as the leakage of air from the base region with an underfed jet will rapidly reduce the base pressure until the pressure differential across the jet is balanced by the momentum flux. Tulin's results showed that if overfeeding of the jet occurred when the GEM was rising, and if underfeeding occurred when it was sinking, then the motion was stable. However, with very large base loadings or a large hollow below the base, the converse could occur and the motion become unstable when the parameter β equaled unity:

$$\beta > 1 \text{ for stability} \tag{1}$$

Eames[3,4]

A very detailed discussion of stability in heave and pitch was given by Eames in Ref. 3 and summarized in Ref. 4. In general, Eames agreed with Tulin's treatment and derived similar results, but made the following observations for the case of constant momentum flux.

1) The time taken for the jet to adjust to changes in pressure distribution is very small and can be neglected.

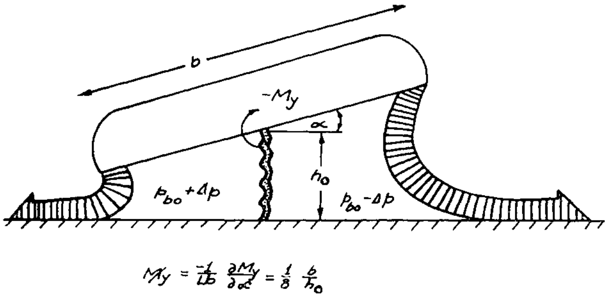

Fig. 2 Eames "membrane" hypothesis giving instability in pitch.

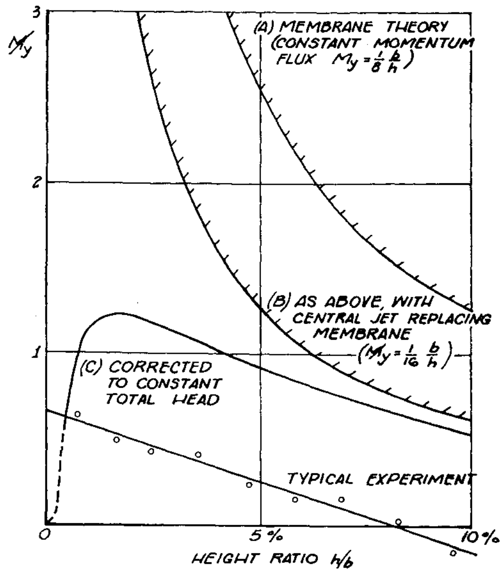

Fig. 3 Typical results for stability in pitch.

2) Provided that β is large, the characteristic cubic equation in heave can be reduced to a subsidence of very short time constant (approximately $1/\beta - 1$) and a quadratic. The frequency of the quadratic is as found by Tulin, and the damping ratio is bivalued according to the condition of the jet. He recommended the use of the harmonic mean of the two damping values as being most accurate for a linear solution.

3) In pitch, Eames considered the case of a "membrane" dividing the base region, as shown in Fig. 2, into two separate sealed chambers, and showed that this gave static stability described by the parameter:

$$\tilde{M}_y = -(\partial M_y/\partial\alpha)/Lb = \tfrac{1}{8}(b/h_0) \tag{2}$$

He also considered the case of a central jet in place of a membrane, and concluded that such a jet, at constant momentum flux, would also give restoring moments by causing a difference of pressure to arise between the two parts of the base, but that, in this case, at the most,

$$M_y = b/16h_0 \tag{3}$$

Similar results were obtained in England by Saunders-Roe.

The writer pointed out that this solution could not possibly agree with experiment, since in most experimental vehicles the fans fed a plenum chamber, which supplied the jets. Hence, although the total mass flow might remain constant, the mass flow and hence the momentum flux of the lower jet would be reduced by the higher average static pressure of the jet orifice, and the momentum flux of the jet would be correspondingly reduced. The stability could be rather more accurately determined by assuming that the total head supplied to the jets remained constant during an oscillation.

Eames' revised theoretical values are given in Fig. 3. It is evident that the general trend is now in better agreement with experiment. (The instability at the greater altitudes can be explained by the presence of cross flow under the base.)

Mankuta[5] and Payne[6]

Mankuta suggested that stability at low altitudes could be greatly improved in a multifan machine by physically compartmenting the plenum chamber to separate the fans (Fig. 4), and Payne independently demonstrated with the Frost Mine-Search-Head-Carrier (MSHC) GEM that separate fans were not necessary; one could effectively compartment a single fan by carrying the partitions up to a point just below

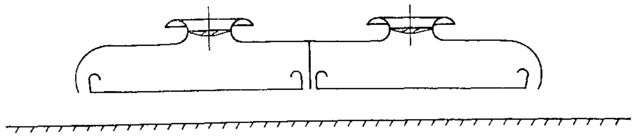

Fig. 4 Bell aerosystems GEM (Mankuta) showing physical compartmentation of plenum feeding annular jet to improve stability at low altitudes.

the fan disk (Fig. 5). Such compartmentation obviously works just as well for a plenum chamber machine as for the annular jet.

Cross,[7] Strand,[8] and Walker[9,10]

The assumption of "constant total head" was applied to the case of the GEM in heave by Cross, who took advantage of Eames' linearization to assume that he would be justified in calculating separately the 'derivatives' or partial differentials of the heave motion, and specifically the force due to rate of change of height through the equilibrium height and the force due to change of height at zero rate of change of height.

Strand made a similar calculation for the "underfed" case only, and Walker attempted to correlate these theories with experiment. However, the results differed wildly and did not agree well with experiment or with the theoretical values recalculated by Walker; thus in 1962 he proposed to Transport Research and Engineering Command (TRECOM)[9] that an attempt should be made to solve the damping problem experimentally by measuring the two derivatives directly and comparing the calculated damping ratio and frequency, including an allowance for the effect of measured fan characteristics with values obtained by oscillating the GEM model.

Payne,[11] and Webster and Lin[12]

In 1963, papers were received from Payne (then with Frost Engineering) and from Webster and Lin of Hydronautics who solved the general problem of the heave stability of annular jet ground effect machine and of plenum chambers both with arbitrary fan characteristics.

However, these papers do not compare in directness with the approach of Cross or Strand, and the second paper continued to emphasize the importance of β while apparently ignoring the fact that an unstable fan characteristic might change the sign of other coefficients in the general equation.

Nay[13] and the Present Paper

Nay, of Hughes aircraft, presented a paper on the Hughes Hydrostreak in which a linear variation of fan pressure with flow was assumed. This paper is of the first example known to the writer of the inclusion of general fan characteristics in the stability equation of a GEM, but unfortunately no derivation of the results was given.

The present paper takes up the methods of Cross and Strand but adds an arbitrary linear variation of jet total head with jet flow. It is gratifying that the result for the plenum chamber agrees with that given by Nay, but the method has severe limitations that are discussed later and is only applicable to cases of mild instability.

Stability of the Annular Jet GEM with Arbitrary Fan Characteristics

Method

Following Cross and Strand, we will assume the general results of Eames and Tulin, which show that the adjustment time of the jets can be neglected and that in the normal case the parameter β is large. (We can then also assume $h = h_b$.)

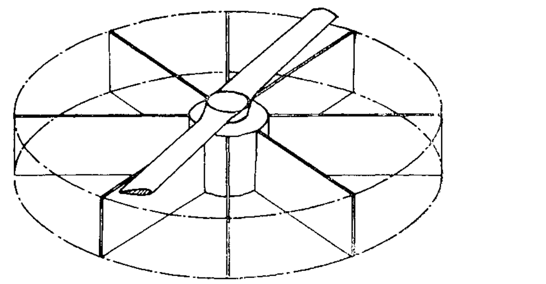

Fig. 5 Physical compartmentation of a single fan (Payne) used on the Frost Mine-Search-Head-Carrier GEM.

This implies that the loadings are low enough so that the air can be regarded as an incompressible fluid, and also that the heave equation of motion of the GEM is a simple linear differential equation of the form

$$W/g\ddot{h} + W/gK\dot{h} + W/g\omega^2(h - h_0) = 0 \tag{4}$$

where

W/g = mass of GEM, slugs
h = height of jet exit above ground plane
h_0 = equilibrium height of jet exit above ground plane
ω = undamped natural frequency in heave

Provided that the damping is low, the latter factor can be taken as

$$\omega^2 = \partial\ddot{h}/\partial h \text{ at } \dot{h} = 0, h = h_0 \tag{5}$$

the static altitude stability.

Fan and Duct Characteristics

Fan characteristics will be assumed to be represented by a plot of the total head rise through the fan P_m vs the volume flow Q. In practice it is the total head of the air supply to the base or base jets that control the lift of the GEM, and this may be appreciably lower than the total head of the fan due to ducting losses.

Hence we will work in terms of P_t the excess total head (relative to atmosphere) available in the jet at the base rather than of P_m, which is the total head at the entry of the duct work (Fig. 6).

Note that as duct work losses commonly are expressed in terms of the velocity head at the fan we may write

$$P_m = P_t + \eta q_m \tag{6}$$

where q_m is the dynamic head at the entry to the diffuser from the fan, and η is the efficiency of the diffuser.

Stanton-Jones quotes a duct and diffuser loss of 40% in a very efficiently designed but complex system (SRN-2), whereas Payne suggests that in the case of our own test model hovering near the ground we have a sudden area expansion of 3·25:1, and the calculated loss is 50% of the dynamic head.

Figure 7 shows the fan characteristics for a small high-pitched commercial fan. The results are expressed in terms of the nondimensional coefficients

$$\lambda \text{ and } C_p \; [= P_m/q_t] \tag{7}$$

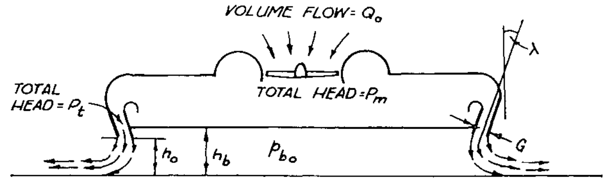

Fig. 6 Geometry of annular jet GEM.

where

q_t = the dynamic head at the tip of the fan

$= \frac{1}{2}\rho V_t^2$

$= \frac{1}{8}\rho\Omega^2 D^2$

so that

$$C_p = 8P_m/\rho\Omega^2 D^2 \quad (8)$$

while

λ = average velocity through fan disk/tip speed of fan

$= Q(\text{per fan})/(\pi/4 \cdot D^2 \cdot \Omega/2 \cdot D)$ (9)

$= 8Q(\text{per fan})/\pi\Omega D^3$

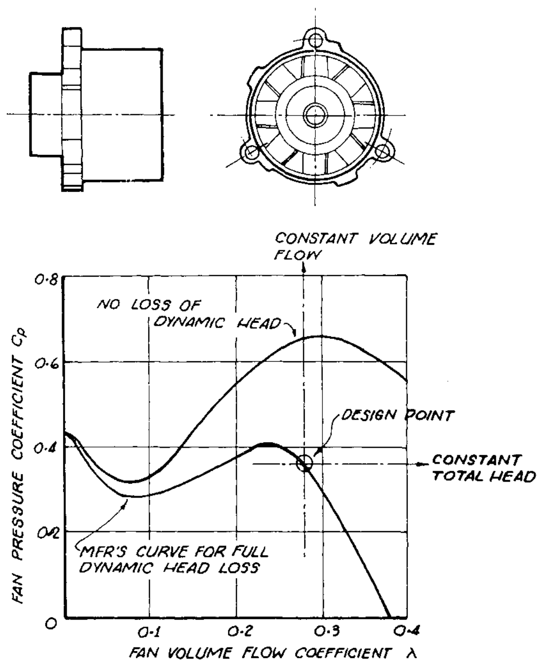

Fig. 7 Characteristics of a high-pitch axial fan (Globe VAX-2-MM)

Now

q_m = average dynamic pressure at entry of plenum

$= \frac{1}{2}\rho$ (average velocity through fan disk)^2

$= \frac{1}{2}\rho[4Q/\pi D^2]^2$ (10)

$= (8\rho/\pi^2).Q^2/D^4$

Alternatively,

$$q_m = \frac{1}{2}\rho[\lambda\Omega D]^2 = \frac{1}{2}\rho\lambda^2\Omega^2 D^2 \quad (11)$$

Thus, to allow for duct losses,

$$C_{pm} = C_{pt} + \eta q_m/q_t = C_{pt} + 4\eta\lambda^2 \quad (12)$$

The lower curve in Fig. 7 shows the published manufacturer's data, which give flow vs static back pressure when exhausting into a large plenum. Assuming that the latter case corresponds to 100% loss, we may derive the dotted curve for the total head available for the fan exit.

Note that in both cases with this high-pitch, high-solidity fan, there is a wide region where the value of C_p increases as λ increases, an unstable characteristic. The design operating condition corresponds to the point where

$$\partial C_p/\partial\lambda = -C_p/\lambda \text{ (peak efficiency)} \quad (13)$$

and the lines for constant total head (C_p = const) and constant volume flow (λ = const) are also shown.

Estimation of the Static Stability Derivative $\partial L/\partial h$ and the Undamped Natural Frequency ω

The following elegant and concise treatment is due to Alastair Anthony. Let us assume that the characteristics of the fan can be completely described by the volume flow, so that the total head in the jet is given by

$$P_t = f(Q) \quad (14)$$

A variation of base pressure will cause a variation in back pressure on the jet and vary both volume flow and P_t. However, this can be calculated by assuming that the volume flow is related to the total head in the jet by a function of the height parameter h/h_0:

$$Q = S_j V_{e0} \cdot \xi(h/h_0) \quad (15)$$

where

$$V_{e0} = [2P_t/\rho]^{1/2}$$

and that the base pressure is also related to total head and height by the equation

$$P_b = P_t \cdot \psi(h/h_0) \quad (16)$$

Now from (15)

$$Q \propto (P_t)^{1/2}\xi$$

$$Q'/Q = \tfrac{1}{2}P_t'/P_t + \xi'/\xi$$

where

$$Q' = \partial Q/\partial(h/h_0) \text{ etc.}$$

$$P_t/Q(\partial Q/\partial P_t)(P_t'/P_t) = \tfrac{1}{2}(P_t'/P_t) + \xi'/\xi \quad (17)$$

$$\frac{P_t'}{P_t} = \frac{\xi'/\xi}{[(P_t/Q)(\partial Q/\partial P_t) - \frac{1}{2}]} = F\left[\frac{\xi'}{\xi}\right] \quad (18)$$

From (16)

$$P_b'/P_b = (P_t'/P_t) + (\psi'/\psi) \quad (19)$$

$$P_b'/P_b = \xi'/\xi \cdot F + \psi'/\psi = \mathfrak{F} \quad (20)$$

and

$$\partial L/\partial h = p_b' S_b/h_0 = (p_b S_b/h_0)[\mathfrak{F}] \quad (21)$$

Now the undamped natural frequency is given by

$$\omega^2 = -g/W \cdot (\partial L/\partial h) \quad (22)$$

$$W = S_b p_{b_0} \quad (23)$$

Therefore

$$\omega = [g/h_0]^{1/2}\,[-\mathfrak{F}]^{1/2} \quad (24)$$

Note that the stability depends not only on F, the fa characteristic, but also on the characteristics of the annula jet. If ψ'/ψ is negative, F could take an appreciative posi tive value before instability occurred.

Estimation of the Damping Derivative $\partial L/\partial \dot{h}$ and K_s for the Underfed Jet: Sinking GEM

We assume that the GEM is sinking at a rate $\dot{h}$ through th equilibrium height. Now if $\beta \gg 1$ as we specified, air will b forced out below the cushion and the jet will be underfed. Hence the jet configuration will correspond to that of another altitude, $h_0 - \Delta h$ and Δh will be proportional to $\dot{h}$. Hence, following Strand, we may write

$$\partial L/\partial \dot{h} = (\partial L/\partial h) \cdot (\Delta h/\dot{h}) \tag{25}$$

but we may also note that $\Delta h/\dot{h}$ will be fixed by the geometry and gross loading, whereas $\partial L/\partial h$ will be affected by the annular jet properties and by fan characteristics that will be included in $\partial L/\partial h$ if calculated as in (33). Now the outflow from the cushion will be $S_b \dot{h}$ and, as the air is incompressible,

$$\dot{h}S_b = \Delta h \cdot CV_{e0} \tag{26}$$

whence

$$\frac{\Delta h}{\dot{h}} = \frac{S_b}{c}\left[\frac{\rho}{2p_{b_0}}\right]^{1/2} \tag{27}$$

since the air will be expelled from the cushion with a total head equal to p_{b_0}:

$$\frac{\partial L}{\partial \dot{h}} = \frac{\partial L}{\partial h}\frac{S_b}{C}\left[\frac{\rho}{2p_{b_0}}\right]^{1/2} \tag{28}$$

and from (21)

$$\frac{\partial L}{\partial h} = \frac{p_{b_0}S_b}{h_0}[\mathfrak{F}]$$

Therefore

$$\frac{\partial L}{\partial \dot{h}} = \frac{p_{b_0}}{h_0}\frac{S_b^2}{C}[\mathfrak{F}]\left[\frac{\rho}{2p_{b_0}}\right]^{1/2} \tag{29}$$

and since

$$K_s = -g/W(\partial L/\partial \dot{h})$$

$$= -(g/S_bP_{b0})(\partial L/\partial \dot{h}) \tag{30}$$

$$= -\frac{Sb}{C}\frac{g}{h_0}[\mathfrak{F}]\left[\frac{\rho}{2p_{b_0}}\right]^{1/2} \tag{31}$$

If

$$K_0 = \frac{S_b}{c}\frac{g}{h_0}\left[\frac{\rho}{2p_{b_0}}\right]^{1/2} \tag{32}$$

$$\bar{K}_s = K_s/K_0 = -\mathfrak{F} \tag{33}$$

Estimation of the Damping Derivative $\partial L/\partial h$ and of K_R for the Overfed Jet: GEM Rising

We now assume that the GEM is rising through the equilibrium altitude and that the jet splits to supply air to the cushion at a rate $S_b \dot{h}$. Note that this airflow, being from the cushion side of the jet, must be supplied at a static pressure of p_{b_0} if displacements are small, and we will assume that turbulent mixing dissipates the momentum of this jet.

The portion of the jet supplying air to the cushion will have a width of ΔG and

$$\dot{h}S_b = C\cdot\Delta G\cdot\left[\frac{2(P_t - p_{b_0})}{\rho}\right]^{1/2} \tag{34}$$

Now the reduced jet width will support a reduced base pressure, equivalent to a gain in altitude of Δh, and we will have

$$\Delta G/G = \Delta h/h_0$$

Hence

$$\partial L/\partial \dot{h} = (\partial L/\partial h)(\Delta h/\dot{h}) \tag{35}$$

$$= (\partial L/\partial h)(\Delta G/G)(h_0/\dot{h})$$

But, from (34),

$$\Delta G = \dot{h}S_b/c[\rho/2(P_t - p_{b_0})]^{1/2}$$

Hence

$$\frac{\partial L}{\partial \dot{h}} = \frac{\partial L}{\partial h}\frac{h_0}{G}\frac{S_b}{C}\left[\frac{\rho}{2(P_t - p_{b_0})}\right]^{1/2} \tag{36}$$

But

$$p_{b_0} = \psi P_{t_0} \text{ and } \frac{h_0}{G} = \frac{1+\sin\lambda}{x}$$

therefore,

$$\frac{\partial L}{\partial \dot{h}} = \frac{\partial L}{\partial h}\frac{(1+\sin\lambda)}{x}\frac{S_b}{c}\left[\frac{\rho}{2p_{b_0}}\right]^{1/2}\left[\frac{\psi}{1-\psi}\right]^{1/2} \tag{37}$$

Substituting for $\partial L/\partial h$ as before, we find

$$K_R = -\frac{g}{S_bp_b}\frac{\partial L}{\partial \dot{h}} \tag{38}$$

$$= \frac{g}{h_0}\frac{S_b}{c}\left[\frac{\rho}{2p_{b_0}}\right]^{1/2}\frac{1+\sin\lambda}{x}\left[\frac{\psi}{1-\psi}\right]^{1/2}\left[-\mathfrak{F}\right]$$

$$\bar{K}_R = \frac{1+\sin\lambda}{x}\left[\frac{\psi}{1-\psi}\right]^{1/2}\left[-\mathfrak{F}\right] \tag{39}$$

$$\frac{\bar{K}_R}{\bar{K}_S} = \frac{1+\sin\lambda}{x}\left[\frac{\psi}{1-\psi}\right]^{1/2} \tag{40}$$

Estimation of Damping Ratios: ζ_S and ζ_R

From the preceding work it is easy to show that

$$\bar{\zeta}_S = \zeta_S/\zeta_0 = [-\mathfrak{F}]^{1/2} \tag{41}$$

$$\bar{\zeta}_R = \frac{\zeta_R}{\zeta_0} = \frac{1+\sin\lambda}{x}\left[\frac{\psi}{1-\psi}\right]^{1/2}\left[-\mathfrak{F}\right]^{1/2} \tag{42}$$

where

$$\zeta_0 = \frac{S_b}{2c}\left[\frac{g}{h_0}\right]^{1/2}\left[\frac{\rho}{2p_{b_0}}\right]^{1/2}$$

Payne and Cross both recommend the arithmetic mean of these two values be used in calculating response.

Discussion of Annular Jet Results for Simple Cases

The fan and ducting factor F appears in both damping and static stability terms, but the actual modifying factor $\mathfrak{F}[= F(\xi'/\xi) + (\psi'/\psi)]$ includes some characteristics of the jet itself.

The motion will be a damped oscillation if $-\mathfrak{F}$ is positive, and a divergence if $-\mathfrak{F}$ is negative. The solution will be a subsidence if ζ is greater than 1.0, but no divergent oscillation can occur.

The solutions are completely general and can be used with any theoretical or practical determination of 1) the fan and duct characteristic F; 2) the jet flow characteristic $\xi(h/h_0)$; and 3) the base pressure characteristic $\psi(h/h_0)$.

Application of Exponential Theory

The result can be most readily appreciated by assuming that the exponential theory applies. This gives results in terms of the nondimensional parameter x, where

$$x = G/h\,(1+\sin\lambda) \tag{43}$$

$$\frac{\partial(\ )}{\partial h/h_0} = -x\frac{h_0}{h}\frac{\partial(\ )}{\partial x} \tag{44}$$

We are interested in small perturbations around the equilibrium condition, so that $h = h_0$

$$\frac{\partial(\ )}{\partial h/h_0} = -x\frac{\partial(\ )}{\partial x} \tag{45}$$

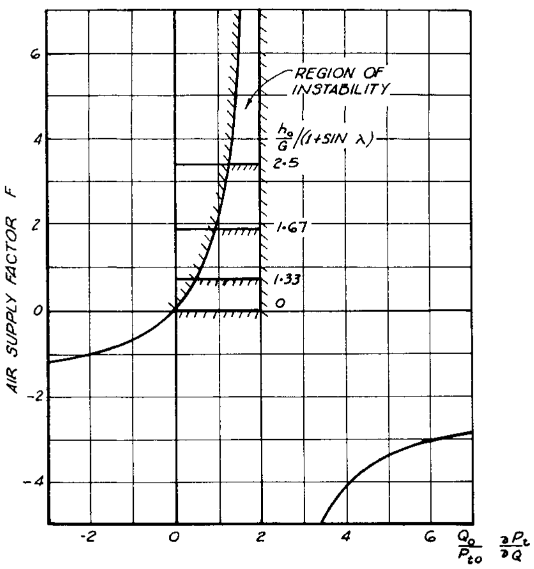

Fig. 8 Permissible variation of the parameter "F" for stability of an annular jet GEM.

The exponential theory gives

$$Q/S_j v_{e0} = (1 - e^{-x}/x) \tag{46}$$

so that

$$\xi'/\xi = 1 - [x/(e^x - 1)] \tag{47}$$

As $x \to 0$ this function tends to $\frac{1}{2}x$ and then to 0. For base pressure

$$p_{b_0}/p_t = 1 - e^{-2x} \tag{48}$$

so that

$$\psi'/\psi = -2x/(e^{2x} - 1) \tag{49}$$

As $x \to 0$ this parameter tends to $\frac{1}{2}x - 1$ and then to -1. The remaining damping parameter is

$$[(1 + \sin\lambda)/x][\psi/(1 - \psi)]^{1/2} \tag{50}$$

which can be written as

$$\frac{2(1 + \sin\lambda)}{[2x]^{1/2}}\left[\frac{\psi}{2x(1 - \psi)}\right]^{1/2} \tag{51}$$

and substituting for ψ, this gives

$$\frac{2(1 + \sin\lambda)}{(2x)^{1/2}}\left[\frac{e^{2x} - 1}{2x}\right]^{1/2} \tag{52}$$

As x tends to zero, this tends to

$$2(1 + \sin\lambda)/(2x)^{1/2} \tag{53}$$

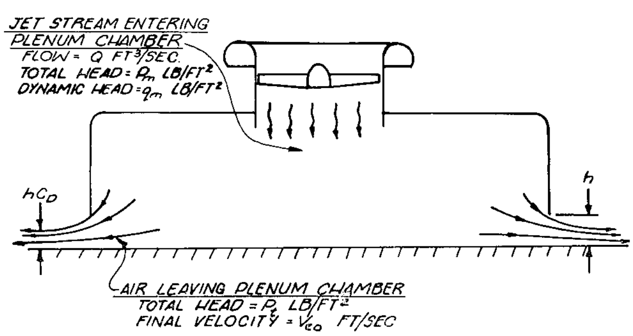

Fig. 9 Flow in a plenum chamber GEM.

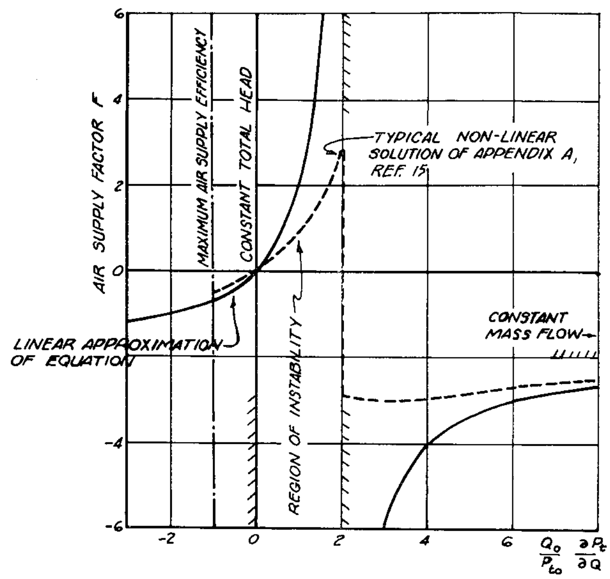

Fig. 10 Variation of the stability parameter "F" with $(Q_0/P_{t0}).(\partial P_t/\partial Q)$.

Permissible Range of F for Stability, Exponential Theory

Applying these results to the calculation of $\mathfrak{F}$ we have

$$\begin{aligned}\mathfrak{F} &= \xi'/\xi F + \psi'/\psi \\ &= \left[1 - \frac{x}{(e^x - 1)}\right]F - \frac{2x}{e^{2x} - 1}\end{aligned} \tag{54}$$

The first coefficient is always a positive, and lies between 0 and 1. The second term is always negative. The critical value of F will occur when $\mathfrak{F} = 0$, i.e.,

$$F_{\text{crit}} = \frac{2x}{(e^{2x} - 1)\{1 - [x/(e^x - 1)]\}}$$

Hence, positive values of F can be permitted without causing $\mathfrak{F}$ to become positive for values shown in Table 1.

Hence, with a really thin jet, almost any fan will be stable, but for $1/x = 1.667$, which is near the optimum for power consumption, the GEM will be unstable if F exceeds 1.91, i.e., when $(Q_0/P_{t0})(\partial Pt/\partial Q)$ lies between 0.9 and 2.0 (Fig. 8).

Stability of the Plenum Chamber with Arbitrary Fan Characteristics

The plenum chamber (Fig. 9) solution can be deduced easily from the previous result for the annular jet. From Fig. 9 we have

$$Q = C_D hCV_{e0}$$

but

$$Q = S_j V_{e0}\xi(h/h_0)$$

Hence

$$\xi(h/h_0) = (C_D C/S_j)\cdot h$$

$$\xi' = (C_D C/S_j)h_0$$

therefore

$$\xi'/\xi = h_0/h$$

and at

$$h = h_0 \qquad \xi'/\xi = 1$$

Similarly

$$p_{b0} = P_m$$

therefore

$$\psi = 1$$

$$\psi' = 0 \text{ and } \psi'/\psi = 0$$

Hence, for this case,

$$\mathfrak{F} = \xi'/\xi(F) + \psi'/\psi$$
$$= F$$

It follows that the undamped natural frequency is

$$\omega^2 = -F[g/h_0]$$

and that the damping ratio

$$\zeta = \bar{\xi}\zeta_0$$

where ζ_0 is as defined for the annular jet GEM, and

$$\bar{\xi} = [-F]^{1/2}/C_D$$

(Note that, for the plenum chamber C_D, the discharge coefficient must obviously be introduced.[15] The fact that the exponential theory gives a discharge coefficient of 1.0 for a thick jet is a failing of this particular annular jet theory.)

Discussion of Plenum Chamber Results for Simple Cases

The equation of motion is

$$\ddot{h} + \frac{2\zeta_0}{C_D}\left[\frac{g}{h_0}\right]^{1/2}(-F)\dot{h} + \frac{g}{h_0}(-F)(h - h_0) = 0 \quad (55)$$

and the equation will be damped oscillation provided that $-F$ is positive and $-F\zeta_0/C_D < 1.0$. However, if $-F$ becomes negative, then the motion will become a divergence. An unstable oscillation cannot occur because the term F occurs in both frequency and damping coefficients.

Constant Total Head

For the constant total head case, $\partial P_t/\partial Q = 0$ and $F = 0$. This case is one of neutral stability; there is no restoring force due to displacement and no damping provided that the fan always supplies a constant total head at the exit regardless of the volume flow requirements.

Constant Volume Flow

F can also be written as

$$\frac{-1}{\frac{1}{2} - (Pt_0/Q_0)(\partial Q/\partial P_t)}$$

which is the fan effect parameter quoted by Nay. It is easy to see that for constant volume flow $\partial Q/\partial P_t = 0$ and $F = -2$. This result will hold as long as $\partial Q/\partial P_t$ is small, whether it is of positive or negative sign.

Peak Efficiency

At peak output efficiency

$$\partial P_t/\partial Q = -P_{t0}/Q_0 \quad (56)$$

For this case $F = -1.0$.

General Case

A general plot of F vs $(Q_0/P_{t0})(\partial P_t/\partial Q)$ is given in Fig. 8 and divergence occurs if $(Q_0/P_{t0})(\partial P_t/\partial Q)$ lies between 0 and +2. However if $(Q_0/P_{t0})(\partial P_t/\partial Q)$ slightly exceeds 2, there is a large stable value of F that seems quite unreasonable.

Payne has considered this case in more detail[14] in a forthcoming report using an analysis sketched briefly in Appendix A of Ref. 15 and shows that in this region the characteristic equation is nonlinear; the effect is to reduce the variation of F. However, there is no doubt that the result for $F = 0$ is correct; hence divergence can certainly occur for (Q_0/P_{t0})-$(\partial P_t/\partial Q)$ between 0 and 2 and will probably take the form of a limit cycle oscillation over the unstable region of the curve (see Fig. 10).

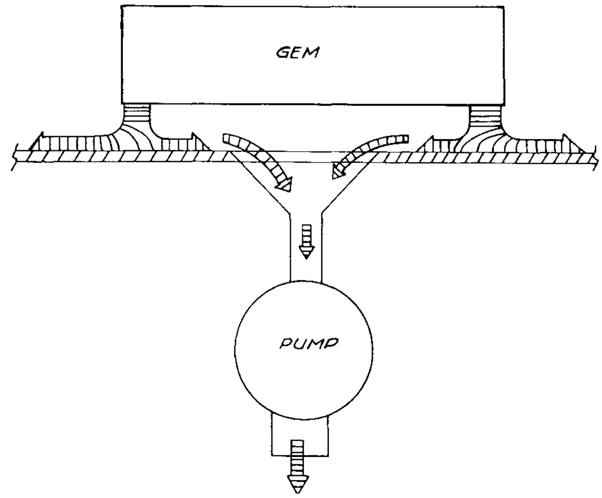

Fig. 11 Arrangement for experimental determination of $L\partial/\partial h$.

A GEM Stability Parameter That is Independent of the Fan Characteristics

It is interesting to note that the general stability equation

$$\ddot{h} + 2\zeta\omega\,\dot{h} + \omega^2(h - h_0) = 0$$

which involves the two experimentally measurable parameters ζ and ω, can also be written

$$\ddot{h} + 2\zeta_0(-\mathfrak{F})[g/h_0]^{1/2}\dot{h} + g/h_0(-\mathfrak{F})(h - h_0) = 0$$

The ratio of the second and third coefficient to the equation is $2\zeta/\omega$ and is independent of $(-\mathfrak{F})$. In fact, substituting for ζ_0,

$$2\zeta/\omega = k(S_b/c)[\rho/2p_{b_0}]^{1/2}$$

where $k = 1/c_D$ for the plenum chamber, and is

$$\left[1 + \frac{1 + \sin\lambda}{(2x)^{1/2}}\right]$$

for a thin annular jet.

For the plenum chamber, therefore, $2\zeta/\omega$ does not vary with h at all, and only varies slowly with h for the thin jet. The parameter is therefore very suitable for investigating practical stability tests, especially since if desired the fan characteristics can be inferred from the static calibration of lift against hover height. Note that $2\zeta/\omega$ is also equal to $(\partial L/\partial\dot{h})/(\partial L/\partial h)$, the ratio of the two stability derivatives.

Comparison with Experiment

A detailed experiment to investigate heave damping was planned by the writer in 1962 and has received the support

Table 1 Maximum permissible values of F, annular jet GEM

$1/x$	=	1.0	1.25	1.67	2.5	5.0
x	=	1.0	0.8	0.6	0.4	0.2
F_{crit}	=	0.75	1.17	1.91	3.41	8.40

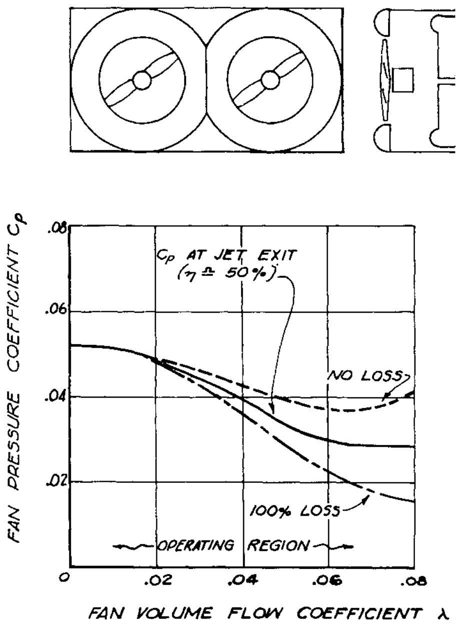

Fig. 12 Experimentally determined flow characteristics for a model GEM.

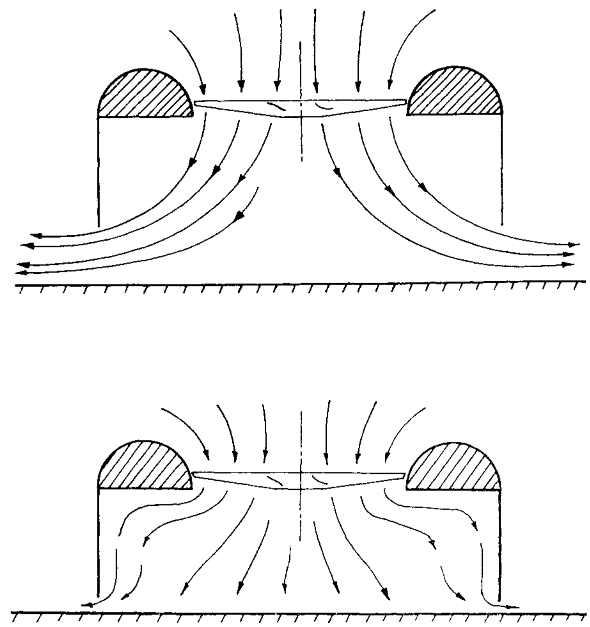

Fig. 13 Two alternative patterns of flow that occurred in a model plenum chamber GEM.

of U.S. Army TRECOM. The experiment was to take an annular jet model and calibrate the lift against altitude, thus determining $\partial L/\partial h$. Then a direct determination of $\partial L/\partial \dot{h}$ would be made by blowing air into or sucking air out of the base region, thus giving a variation of lift (Fig. 11). Finally, the damping and frequency would be measured from oscillation tests and compared with the calculated results using $\partial L/\partial h$ and $\partial L/\partial \dot{h}$.

The fan characteristics would be determined and used to check the theoretical estimates of $\partial L/\partial h$ and $\partial L/\partial \dot{h}$. The fan calibration is given in Fig. 12, from which we can at once see that no instability should occur, and none was recorded with the annular jet model.

Detailed results are not yet available but, in general, the theoretical predictions are borne out, and the term $\partial L/\partial \dot{h}$ does appear to account for at least 80% of the total damping. No attempt has yet been made to analyze the results in terms of the parameter $2\zeta/\omega$.

Limit oscillations were observed with the plenum chamber model, but these were not due to instability of the fan characteristics themselves, but apparently to a sudden change of flow pattern in the plenum chamber itself (Fig. 13).

Conclusions

This analysis is restricted to small linear oscillations, but still demonstrates conclusively that an allowance for the fan and ducting characteristics must be included in any future estimates of GEM response characteristics. Such experimental evidence as is available confirms that the physical basis for the theory is essentially correct.

The analysis is equally applicable to the calculation of cushion derivatives in pitch and roll, since these cases are essentially extensions of the heave case. The possibility of adjusting the "ride" characteristics of a GEM by altering the characteristics of the air supply system may be extremely useful in the design of future large GEMs, which will tend to be overdamped and hard riding. It may even be possible to adjust the damping in this way during a run to suit the sea conditions.

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